

弧齿锥齿轮切齿和啮合过程的数字仿真*

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【摘要】 针对弧齿锥齿轮的切齿和啮合过程,建立了用数字仿真技术研究锥齿轮齿面形成和轮齿啮合过程的方法。在该方法中,将齿面切制时的线共轭条件转化为约束极值问题,据此获得被切齿面的数值模型;将轮齿啮合时的点共轭条件转化为在两齿面上求距离最近的点,并借助齿面数值模型,获得接触印痕和传动误差。用本文方法进行了某航空弧齿锥齿轮的切齿和啮合过程的数字仿真。

主题词: 数字仿真 弧齿锥齿轮 共轭 曲面

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1 前言

弧齿锥齿轮切齿和啮合过程的研究,不但提高了齿轮传动质量,而且丰富了共轭曲面理论^[1,2]。已有研究大多是以刚体运动几何学为基础,建立的齿面啮合基本条件是 $\vec{v}^{(12)} \cdot \vec{n} = 0$,但在弧齿锥齿轮实际啮合过程中,由于齿面失配、安装误差及受载变形等因素的影响,条件 $\vec{v}^{(12)} \cdot \vec{n} = 0$ 有时会不成立(如在非共轭点或边缘接触点处);此外,针对解析形式齿面所建立的齿面接触分析方法(TCA和LTCA),难以用于测绘所得的或以网格给定的数值形式齿面。本文将共轭曲面数字仿真技术^[3,4]用于研究弧齿锥齿轮的切齿和啮合,以约束极值方程取代啮合条件,形成齿面的数值模型,在此基础上,建立了齿轮啮合性能分析的数值方法。

2 切齿仿真

弧齿锥齿轮被切齿面与铣刀盘形成的产形轮齿面之间为线共轭曲面。用 Σ_1 和 Σ_2 表示小轮和大轮的被切齿面,用 Σ_F 和 Σ_P 表示相应的产形轮齿面。下面以加工右旋小轮凹面为例,说明切齿仿真原理。

在机床摇台上建立固定坐标系 $O_F\{O_F, X_F, Y_F, Z_F\}$,在齿轮1上建立动坐标系 $O_1\{O_1, x_1, y_1, z_1\}$,如图1所示。在 O_F 中,产形轮齿面点径矢和法矢分别为 $r_F = [X_F, Y_F, Z_F]^T$ 和 $n_F = [n_x, n_y, n_z]^T$,其中各分量为:

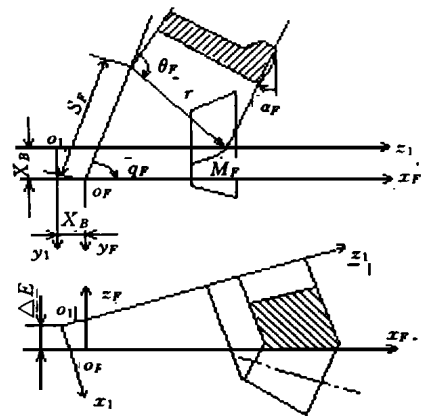


图1 小齿轮切齿坐标系

$$\begin{cases} x_F = u_F \sin \alpha_F \cos(\theta_F - q_F + \varphi_F) + s_F \cos(q_F - \varphi_F) \\ y_F = u_F \sin \alpha_F \sin(\theta_F - q_F + \varphi_F) - s_F \sin(q_F - \varphi_F) \end{cases}, \quad z_F = r_o \operatorname{Ctg} \alpha_F - u_F \cos \alpha_F \quad (1)$$

$$n_x = \cos \alpha_F \cos(\theta_F - q_F + \varphi_F), \quad n_y = \cos \alpha_F \sin(\theta_F - q_F + \varphi_F), \quad n_z = \sin \alpha_F \quad (2)$$

式中: u_F 和 θ_F 为 Σ_F 的曲面参数; r_o 和 α_F 为刀盘半径及压力角; s_F 和 q_F 为径向刀位及刀位极角; φ_F 为产形轮转角。在 σ_1 中, 小轮齿面点径矢为: $[x_1, y_1, z_1]^T = [R_1 \sin \tau_1, R_1 \cos \tau_1, L_1]^T$ 式中: R_1 和 L_1 为齿面点对其轴线的半径及由节锥顶点沿轴线计量的坐标; τ_1 为齿面点与其在参考面中的映射点间的圆心角。

将 Σ_F 的径矢 r_F 和法矢 n_F 由 σ_F 变换到 σ_1 中, 记作 $r_F^{(1)} = M_{1F} r_F$, $n_F^{(1)} = M_{1F} n_F$ 式中: M_{1F} 为坐标变换矩阵, 其元素值是小轮转角 φ_F 的函数。

在 Σ_F 与 Σ_1 的切齿啮合点处, 径矢相等, 即 $r_1 = r_F^{(1)}$ 。此外, 对于给定的一组 (R_1, L_1) , 齿面点应是参考点距离最近的点, 即在式(1)的条件, 求 τ_1 关于 φ_F 的最小值, 用公式表示为:

$$\tau_1 = \min_{\varphi_F} \arctg[x_F^{(1)} / y_F^{(1)}] \quad (3)$$

用数学规划求解此约束优化问题, 即可获得 Σ_1 的数值模型。

3 啮合仿真

依据两齿轮的工作位置, 将大轮齿面 Σ_2 置于小轮坐标系 σ_1 中, 即将大轮齿面点径矢 r_2 和法矢 n_2 变换到 σ_1 中, 并记作 $r_2^{(1)}$ 和 $n_2^{(1)}$ 。设齿面 Σ_1 和 Σ_2 各绕其轴线 z_1 和 z_2 回转 β_1 和 β_2 角后, 计算基点法矢相等, 即关于 β_1 和 β_2 求

$$\min \|M(\beta_1, z_1) n_1 - M(\beta_2, z_2) n_2^{(1)}\| \quad (4)$$

式中: $M(\beta_1, z_1)$ 和 $M(\beta_2, z_2)$ 为坐标旋转变换矩阵。

若此时两轮齿面计算基点啮合, 则满足条件: $M(\beta_1, z_1) r_1 = M(\beta_2, z_2) r_2^{(1)}$ 。否则由下式求得大轮相对小轮的调整值 H 、 J 和 V

$$\min \|M(\beta_1, z_1) r_1 - M(\beta_2, z_2) r_2^{(1)} - H z_1 - J z_2 - V z_1 \times z_2\| \quad (5)$$

这样求得的 H 、 J 和 V , 就能保证切齿计算基点成为齿轮传动时的啮合点, 而且是齿面接触印痕的中点。为了获得完整的接触印痕, 给定主动轮 1 转角增量 $\delta\beta_1$, 在两齿面 Σ_1 和 Σ_2 上求距离最近的点, 即为该时啮合点, 用公式表示为:

$$\min \|M(\beta_1 + \delta\beta_1, z_1) r_1 - M(\beta_2 + \delta\beta_2, z_2) r_2^{(1)} - H z_1 - J z_2 - V z_1 \times z_2\| \quad (6)$$

求解可得两轮齿面啮合点 (L_1, R_1, τ_1) 和 (L_2, R_2, τ_2) , 以及从动轮 2 的转角 $\delta\beta_2$ 。重复该过程, 即可获得齿面接触迹线, 此迹线上的点即为瞬时接触椭圆中心。

以每个瞬时啮合点为原点, 在齿面上建立局部坐标系 $\{x, y, z\}$, 其中 z 轴与啮合点法矢重合, 利用齿面的数值模型, 将两轮齿面在啮合点处进行二次曲面拟合, 即将两齿面近似表示为:

$$z = 1/2 K_x^{(1)} x^2 + G^{(1)} xy + 1/2 K_y^{(1)} y^2 \quad (7)$$

$$z = 1/2 K_x^{(2)} x^2 + G^{(2)} xy + 1/2 K_y^{(2)} y^2 \quad (8)$$

式中: $K_x^{(1)}$ 、 $K_y^{(1)}$ 、 $G^{(1)}$ 和 $K_x^{(2)}$ 、 $K_y^{(2)}$ 、 $G^{(2)}$ 分别为齿轮 1 和齿轮 2 的齿面法曲率及扭曲率。其值皆由齿面点坐标进行二阶数值微分获得。设齿轮在啮合时, 两齿面间的法向接触变形量为 Δ , 则由式(7)和(8)得瞬时接触椭圆在切平面上的投影方程为:

$$\delta K_x x^2 + 2\delta G xy + \delta K_y y^2 = 2\Delta \quad (9)$$

式中: δK_x 、 δK_y 及 δG 为两啮合齿面间的相对法曲率及相对扭曲率。由该式即可求得接触椭圆的长、短半轴 a 和 b 及长轴与 x 轴间的夹角 τ 。所有瞬时接触椭圆所形成的区域即为接触印痕。

在弧齿锥齿轮啮合过程中, 由于齿面失配和接触变形等因素的影响, 当主动轮 1 作等速转动时, 从动轮 2 作变速转动, 其转角误差为: $\epsilon = \delta\beta_2 + \Delta y/R_2 - \delta\beta_1/i_{12}$ 式中: Δy 为法向接触变形 Δ 沿 y_2 轴的分量, i_{12} 为齿轮副的平均传动比。以小轮转角 $\delta\beta_1$ 为横坐标, 以 ϵ 为纵坐标, 即得齿轮传动的误差曲线。这样就实现了弧齿锥齿轮的啮合特性的数字仿真。

4 仿真实例

用本文所建立的方法, 对某航空弧齿锥齿轮的切齿和啮合过程进行了仿真。该齿轮副的主要参数为: $z_1 = 35$, $z_2 = 47$, $m = 2.875 \text{ mm}$, $\beta_n = 35^\circ$, $\alpha_n = 20^\circ$; 用 SGM 法进行切齿调整计算。

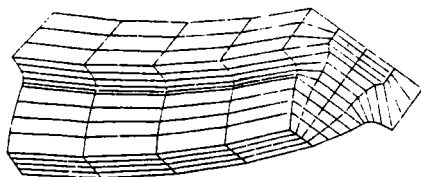


图 2 小轮齿轮网格图

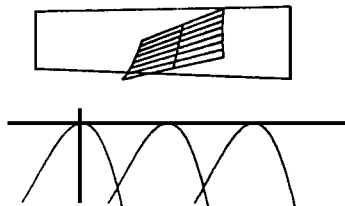


图 3 啮合特性仿真结果

图 2 是依据小轮齿面的数值模型画出的轮齿网格图。图 3 是齿轮传动的啮合特性仿真结果, 其中上图为大轮凸面的接触印痕, 下图为齿轮传动的误差曲线。

5 结 论

(1) 利用仿真技术, 可对弧齿锥齿轮的切齿和啮合过程进行直接描述, 形成齿面数值模型, 并在此基础上进行齿轮副的啮合特性分析。

(2) 仿真方法中用约束优化问题取代了啮合条件 $v^{(12)} \cdot n = 0$, 故对非共轭点或边缘点的啮合分析较为方便。

(3) 仿真方法求齿面接触椭圆时, 用数值微分获得齿面结构参数, 避免了求解共轭曲面理论中的繁杂公式。

(4) 切齿仿真所形成的齿面数值, 可直接用于有限元或边界元分析时的网格, 从而可进行齿轮传动的各种应力计算。

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Key Words Noise propellers Fuselages Effect Noise measurements

COMPARISON BETWEEN TWO KINDS OF CONVENTIONAL LABYRINTH

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ABSTRACT Straight- through and stepped labyrinths are often used in aircraft engine. According to large amounts of experimental data, the empirical formulas of mass flow factor for these two kinds of labyrinth have been provided which are superior to their theoretical formulas due to various corrections. The seal characteristics of the two kinds of labyrinth were compared and analyzed in detail so that their features and reasonable conditions of their application were found out. Finally, the new empirical formulas for calculating their critical pressure ratio were derived too.

Key Words Sealing Flow coefficients Leakage

MULTISECTION LINERS FOR NOISE REDUCTION OF COUNTER- ROTATING FAN

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ABSTRACT It has been found that multisection liners not only attenuate the sound waves in ducts, but also to suppress sound wave through the wave reflection at impedance discontinuities. As a consequence, the maximum noise reduction in a duct of given length can be realized by designing each section liner and different combination. In the present investigation, different combinations of multisection liners have been designed to suppress the discrete noises generated by a counter- rotating fan. The experimental results testified that, the noise was reduced by 9 dB in far field under the condition of 60 cm liner length.

Key Words Fans Perforated plate sound absorption Noise reduction

DIGITAL SIMULATION ON TOOTH CUTTING AND MESHING OF SPIRAL BEVEL GEARS

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ABSTRACT The conventional methods for determining the tooth geometry and tooth contact of spiral bevel gears are based on differential geometries of rigid body movement, and consequently make description of tooth surface and analysis of tooth contact behavior very complicated. In this paper, the authors work out a digital simulation method for tooth cutting and meshing of spiral bevel gears. A numerical model of tooth surfaces is set up by digital simulation of tooth cutting. The analysis of tooth contact behavior is completed with the numerical model of tooth surfaces. Solution of the meshing equation $\mathbf{v} \cdot \mathbf{n} = 0$ in conventional methods is replaced by minimizing the normal distance between points on the tooth

surfaces in our method. The results of digital simulation for the spiral bevel gears used in a Chinese aeroengine show that the presented method describes the tooth shape and tooth contact characteristics more conveniently and efficiently, and can determine optimal cutting adjustment parameters for obtaining spiral bevel gears with good contact quality.

Key Words Digital Simulation Spiral bevel gears Curved surface

FRETTING FATIGUE OF DOVETAIL JOINT UNDER COMBINED CYCLIC LOADING

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ABSTRACT The state-of-art in research on fretting fatigue of dovetail joint in aircraft engine at home and abroad was sized up. Stress analysis of dovetail joint part was carried out. Experimental equipment was designed and set up. A number of tentative experiments was conducted. It is confirmed that the experimental technique and equipment can offer a great deal of data for designing.

Key Words Tenon Cyclic Loads Fatigue

PRESSURE AND AVERAGED THRUST OF A PULSED DETONATION ENGINE MODEL

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ABSTRACT A mode of Pulsed Detonation Engine (PDE) has been made. Its pressure and averaged thrust were analysed and calculated according to detonation wave theory, and then were measured with two sensors in testing of the PDE model. The comparison of the calculated results with the measured data shows that the PDE model can produce the pulsed detonation wave and thrust.

Key Words Impulsive burst Engines Pressures Thrust

EFFECTS OF FILM COOLING ON INTERNAL HEAT TRANSFER COEFFICIENTS OF BLADES

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ABSTRACT Effects of film cooling on the internal heat transfer coefficients of turbine blades were investigated numerically and the key factors have been found out as the film cooling hole distance ratio and the inlet Reynolds number. The local heat transfer coefficient variations were discussed both for the pressure and the suction sides of blades. Numerical results showed that, for the pressure side the internal heat transfer coefficient was much enhanced and could be doubled, as compared with the no-hole passages at the same range.